84(3), 519-534. http://dx.doi.org/10.1007/s10670-017-9969-0

Quantity Tropes and Internal Relations

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This *his is a post-peer-review, pre-copyedit version of an article published in Erkenntnis. The final authenticated version is available online at:*

10.1007/s10670-017-9969-0

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1. Introduction

Trope bundle theories of substance (e.g. Williams 1953, Campbell 1990, Simons 1994, Denkel 1996, Maurin 2002, Giberman 2014) aspire to a construction of objects by means of tropes. According to these nominalist one- or two-category ontologies, tropes form the sole fundamental categories of entities.[1] As nominalist category systems, these theories must avoid explicitly assuming – or implicitly smuggling in – real generality, such as property or relation universals. Tropes are particulars: the *-e* charge of a specific electron is just as particular as the

[1] Some trope theorists (such as Maurin (2002) and Simons (2003)), introduce relational tropes as a category additional to property tropes.

particle whose charge it determines by being its proper part. However, although tropes themselves are determinate particular natures according to standard trope nominalism, they fall under determinables, such as *electric charge*, and determinates, such as *-e charge*. *Prima facie*, the tropes falling under a determinate are exactly similar and the tropes falling under a determinable resemble each other more or less closely. It is a central challenge for any trope bundle theory to give a nominalistically acceptable account of the falling of tropes under determinates and determinables.

In this article, we adopt what Campbell (1990, pp. 24-26) calls a *sparse theory of tropes*. According to this kind of conception, all known tropes are discovered empirically. Moreover, the best examples of monadic tropes are determinate quantities falling under the fundamental physical determinables (such as electric charge, mass, colour charge, spin, etc.). The tropes of some quantity determine intrinsic exact similarities between basic particles and contribute to accounting for their division into natural kinds (such as electron or down quark). Determinate and determinable physical quantities also figure in the laws of nature (such as Coulomb's law) spelling out the causal powers of objects. Here, the trope bundle theorist seems to face a formidable challenge: she is obliged to provide an account of the objective (language- and mind-independent) divisions among tropes into determinable and determinate kinds consistent with the nominalist claim that there are only particulars (tropes and the objects they constitute).

This article aims to address the challenge directly; we spell out how tropes themselves, as determinate particular natures, give rise to their own division into instances of distinct determinate and determinable kinds. The notion of internal relation plays a central role in this explanation. We begin by formulating the distinction between *basic* and *derived internal relations* in Section 2. This distinction aims to be sensitive to the decisive differences between different kinds of internal relations, and – unlike the current standard conceptions of internal relations – it is capable of spelling out the conditions in which an internal relation holds in virtue of the existence of its relata. In Section 3, we argue that quantity tropes are, as determinate particular natures, mutually related by the basic internal relations of proportion and order. Because tropes mutually connected by these internal relations constitute distinct non-overlapping groups, tropes are also instances of distinct determinables and determinates.

In Section 4, we argue that the main earlier attempts at answering the challenge are unsuccessful. According to what we call the "standard view", exact similarity and inexact resemblance are primitive internal relations between tropes. Nevertheless, it remains unclear whether an advocate of the standard view can avoid assuming determinable universals. The other proposed ways of answering the challenge, while avoiding the assumption of universals, lead to serious difficulties, too. Finally, Section 5 concludes the paper and points to certain important issues left open by this discussion.

2. Basic and derived internal relations

The view argued for here assigns a central theoretical role to specific types of relations, namely the relations of *proportion* and *order* between tropes. It is crucial to our view that these relations are *internal*: they constitute no addition of being; in other words, they are not entities in addition to their relata. Since internal relations are not entities of any sort, their holding of their relata should be based entirely on these relata. For the present purposes, it is instructive to begin with two different conceptions of internal relations present in the recent literature.[2]

[2] Ehring (2011, p. 179) employs Armstrong's (1989a, pp. 43-44) characterization of internal relations. Since Armstrong's characterization can be considered a disjunction of [IR1] and [IR2] below, we will not discuss it here.

According to the first conception, a relation is internal if and only if its holding is wholly grounded in the *corresponding properties* of its relata (Wieland & Betti 2008, p. 517). Moreover, the corresponding properties are assumed to be contingent to the relata. As an illustration, and as an explanation of the idea of a "corresponding property", one may take up some monadic properties of macroscopic objects, such as being 1 kg in mass or having a round shape. If object *a* has the property of being round and *b* has that same (or, exactly similar) property, *a* and *b* are related by *having the shape as*. Here, the relation of *having the same shape as* is wholly grounded in the corresponding property of having a round shape (or, corresponding tropes), a property (tropes) possessed by each relatum. Trope theorists and all advocates of property universals take (at least certain) properties as entities. By contrast, there is no such entity as the relation of *having the same shape as* in the world, but only the corresponding properties of *a* and of *b*.[3]

The first conception of internal relations *presupposes* that objects have their monadic natural properties contingently. Internal relatedness is not derivative from the *essential* or *de re* necessary properties of the relata because this conception rules out essential properties as additional entities.[4] Moreover, the grounding of an internal relation by the properties of the

[3] Cf. Campbell (1990, pp. 111-113), Vallicella (2002, p. 5), and Wieland & Betti (2008, p. 517) for similar examples. Campbell and Wieland & Betti do not restrict the corresponding properties to the monadic properties of the relata, but we do not discuss the other options in this article.

[4] Essential properties as further entities are ruled out, e.g. by Armstrong (1989b, 1997) and Campbell (1990). The advocates of the first conception seem to follow this lead. An advocate of the first conception can admit that entities have their formal features (such as particularity and individuality) necessarily, but they are not considered as further entities.

related entities is cashed out in terms of necessitation: necessarily, if the related objects have certain monadic properties, then they are related by the corresponding internal relation. Since the related objects do not necessitate the holding of the internal relation, an internal relation is "wholly grounded" in the properties of the relata:[5]

> [IR1]: Objects *a* and *b* are internally related by relation R if and only if the holding of R is necessitated by the (monadic) properties of *a* and *b*, but the holding of R is not necessitated by the existence of *a* and *b*.[6]

According to the second conception of internal relations, internal relations hold only in virtue of the nature of the related entities themselves, not in virtue of some entities distinct from their relata, such as monadic properties.

Mulligan (1998, p. 344) subscribes to the second conception. One of his examples is the relation of *greater than* holding between two particular "happinesses" (happiness tropes), those of Mary and Erna, when Mary is happier than Erna. These particular happinesses "necessitate the obtaining of the greater than relation between them" (*ibid*. pp. 344-345). Mulligan is explicit in *denying* internality in cases like our two round objects above. While the

[5] Campbell's (1990, p. 112) "*founded external relations*" provide a variant of the same conception of internal relations. Cf. also Maurin (2002, pp. 90-91) and Wieland & Betti (2008).

[6] In both [IR1] and [IR2], the relata of internal relations are called "objects". The relata of internal relations in [IR2] could be entities of any kind. By contrast, in [IR1], the reference to objects (in the sense of bearers of contingent properties) is indispensable. The account of such entities depends on the details of the specific trope bundle theory.

greater than relation between Mary's and Erna's happiness tropes is internal, the relation of *happier than* between Mary and Erna themselves is external. Campbell (1990, p. 37, 112) gives internal relations exactly the same modal characterization as Mulligan, the relations of exact and less than exact resemblance between tropes serving as his prime examples.[7]

Thus, Campbell's and Mulligan's conception is contrary to [IR1]: it is precisely the existence of the related objects that necessitates internal relatedness. This conception could be summarized as follows:

> [IR2]: Objects *a* and *b* are internally related by relation R if and only if the holding of R is necessitated by the existence of *a* and *b*.

The original intention of the advocates of [IR2] is to cash out the grounding of an internal relation in terms of metaphysical necessitation. Nevertheless, if we lift the above-mentioned ban on metaphysically necessary properties, [IR2] classifies relation R as internal in at least two types of cases. First, internal relation R may hold in virtue of the essential or *de re* necessary properties of the relata (let us call these cases of type (i)). Second, internal relation R may hold in virtue of the existence of the relata *simpliciter*, not in virtue of their properties, even essential ones (let us call these cases of type (ii)).

Trope theorists need to introduce internal relations of type (ii). First, tropes stand in some basic formal ontological relations (e.g. distinctness, ontological dependencies, and

[7] Although giving internal relations a Moorean characterization ("internal relations are essential to the identity of their (both) relata") (Moore 1960), Campbell (1990, p. 112) considers this equivalent to the modal characterization. Cf. Maurin (2002, pp. 87-89).

mereological relations) in virtue of the mere existence of the entities.[8] Second, if we postulate tropes or property universals, they are – at least *prima facie* – exactly or inexactly similar in virtue of being specific thin natures (Maurin 2002, pp. 88-89; Simons 2003). However, it is not a reasonable move to rule out internal relations of type (i): a trope theorist may want to accept tropes necessary to objects. According to one such view, if an object exists, it has some of its tropes as its *de re* necessary proper parts.[9]

Thus, there is a need to distinguish between internal relations that hold in virtue of the mere existence of their relata, on the one hand, and internal relations that hold in virtue of the existence of some entities distinct from their relata (such as their contingent or necessary properties), on the other. *Prima facie*, the tropes falling under the same determinable are in the internal relations of "similarity", which hold because tropes are the particular natures they are. Nevertheless, clause [IR2] is insufficient to spell out the conditions that enable these internal relations to hold in virtue of the existence of their relata as contrasted with internal relations holding in virtue of the existence of some entities distinct from the relata. The above distinction between internal relations of type (i) and (ii) does not help us either because the distinction collapses in the case of single, unaccompanied tropes allowed for by Campbell (1990), for instance. All trope bundle theories aspire to *analyse* inherence (objects having tropes), e.g. by means of parthood, co-location, and/or existential dependencies. Campbell (1990) analyses inherence in terms of parthood and co-location: trope *t* is a property of *i* if an only if *t* is a part of *i* and *t* is co-located with *i*. In Campbell's analysis of inherence, unaccompanied tropes inhere in themselves by being necessary properties of their own.

[8] Arguably, any ontological category system must introduce formal ontological relations to avoid infinite regresses (cf. Smith & Mulligan 1983; Simons 2003; Lowe 2006, Sec. 3).

[9] Cf. Keinänen & Hakkarainen 2010; Keinänen 2011, Sec. 4.

In order to capture the conditions in which internal relations hold in virtue of the existence of their relata, we wish to introduce a distinction between two different kinds of internal relation, *basic* and *derived*. Assume that relation R holds of entities $a_1, \ldots, a_n$.[10] A definition of basic and derived internal relations (respectively, [BIR] and [DIR] below) can be framed in terms of a more basic concept, that of *proto internal relation* [PIR]:

> [PIR]: Necessarily, entities $a_1, \ldots, a_n$ are related by *proto internal relation* R if and only if $a_1, \ldots, a_n$ exist.
>
> [DIR]: Necessarily, entities $a_1, \ldots, a_n$ are related by *derived internal relation* R if and only if the holding of R of $a_1, \ldots, a_n$ is derivative from proto internal relations holding between entities some of which are distinct from $a_1, \ldots, a_n$.
>
> [BIR]: Necessarily, entities $a_1, \ldots, a_n$ are related by *basic internal relation* R if and only if R is a proto internal relation and the holding of R of $a_1, \ldots, a_n$ is not derivative from proto internal relations holding between entities some of which are distinct from $a_1, \ldots, a_n$.

The two types of internal relation, basic and derived, are mutually exclusive and jointly exhaustive. All *basic* internal relations are proto internal. *Some* derived internal relations may be proto internal, but some of them are not proto internal because they hold contingently.

[10] This assumption is made in order to make it explicit that in clause [PIR], we restrict to relations R holding of entities $a_1, \ldots, a_n$. An alternative definition of proto internal relations would be a clause like [IR2]: entities $a_1, \ldots, a_n$ are related by proto internal relation R iff, necessarily, R holds of $a_1, \ldots, a_n$ if and only if $a_1, \ldots, a_n$ exist.

All internal relations in the sense of [IR2] are proto internal on the plausible assumption that the holding of an internal relation requires that its relata exist. Moreover, it can be immediately seen from the above definitions that no internal relation in the sense of [IR1] – i.e. an internal relation grounded by the contingent properties of its relata – is proto internal or basic internal. Among proto internal relations, there is a finer distinction based on whether or not the holding of a relation is derivative from the holding of proto internal relations between entities some of which are distinct from the relata of the original relation. Proto internal relations of the latter sub-type – i.e. *basic internal relations* [BIR] – include most of the standard examples of internal relations, such as primitive formal ontological relations and internal relations holding due to the nature of property universals or tropes (cf. above). Basic internal relations obtain due to the existence of the relata themselves. In order for them to hold, there need not be any specific entities distinct from the relata (such as monadic properties of the relata).

Proto internal relations of the former sub-type are derived internal relations: their holding is derivative from proto internal relations holding between entities some of which are distinct from the relata of the original relation. Salient examples of derived proto internal relations are those holding in virtue of the necessary or essential properties of the relata provided that necessary properties are entities distinct from the relata. For instance, two electrons have exactly the same charge because of having two exactly similar charge tropes as their necessary properties. Here, the holding of the internal relation of *having the same charge as* is derivative from the holding of exact similarity between the charge tropes and the holding of whatever proto internal relations accounting for the inherence of these charge tropes in electrons.[11]

In our account, the proto internal relations between objects holding due to self-inherence – *having the same charge as* between two free-floating Campbellian -e charge tropes, for instance – are classified as basic internal. This is, of course, as it should be: in Campbell's theory, individual tropes are co-located parts of their own (limiting cases of inherence). The only relevant factor here for the holding of the internal relation is their exact similarity.

We do not consider the notion of *derivativeness* at work in our definitions as a precise technical concept. In the present paper, we will not take up the challenge of spelling out the details of the relevant kind of derivativeness. The notion is purported to be somewhat loose and theory-neutral, awaiting a more detailed theoretical account. There are, however, two important constraints on any suitable conception of derivativeness: first, *derivativeness* is modally as strong as *metaphysical necessitation* – a set of basic internal relations necessitates the holding of a derived internal relation. Second, the notion of derivativeness must be *hyperintensional*. A merely intensional notion, whose fineness of grain only reaches the level of metaphysical necessity, would be incapable of distinguishing between two different kinds of cases: (1) those in which an internal relation holds in virtue of some entities that are necessarily co-existent with but distinct from the relata (such as their necessary proper parts); and (2) those

[11] In Section 3, we replace exact similarity with the relation of order as a primitive basic internal relation between tropes. Internal relations in the sense of [IR1] are derived internal relations, which can be shown by means of a similar example. For reasons of space, we will leave the discussion of these derived internal relations for another occasion.

in which an internal relation holds in virtue of the mere existence of the relata. In (1), we are dealing with a derived internal relation, whereas in (2), we have a basic internal relation.

The distinction between basic and derived internal relations outlined in this section is capable of specifying the conditions in which an internal relation holds in virtue of the existence of its relata. Most importantly, the notion of basic internal relation also captures the sense in which the relations of proportion and order between tropes – the relations playing a main role in the following sections – are internal.

3. Tropes connected by the relations of proportion and order

The main thesis of this section is that quantity tropes are arranged into natural kinds (i.e. groups of different kinds of tropes) because of being in two different types of necessary relation: the *relations of proportion* and the *relation of order*.[12] Tropes are certain thin particular natures: particular determinate masses, electric charges, and spin quantum numbers, for instance. We will argue that the status of tropes as particular natures has two important consequences with regard to the relations of proportion and order. First, the relations of proportion and order are basic internal relations between tropes; given that certain tropes exist, they are arranged in the relations proportion and order. We need not introduce any additional entities, e.g. property kind

[12] Cf. Bigelow & Pargetter (1990, pp. 55-62) for a similar suggestion to use the relations of proportion to spell out the relations between determinate quantities. However, Bigelow & Pargetter introduce proportions as second-degree relation universals, i.e. relations between relation universals. By contrast, proportions are considered here as *basic internal relations* between quantity tropes.

universals. Second, these relations generate natural divisions among tropes and remain invariant irrespective of the conventional choice of the unit of *any* quantity.

Most of the basic monadic quantities come in natural units (are quantized) and have both positive and negative values. The best-known example is electric charge, which has both negative (-e, -e/3, -2e/3) and positive (e, e/3, 2e/3) basic units.[13] Of the basic monadic quantities, only mass does not come in basic units and has only positive values. To illustrate our account, let us now consider electric charge tropes. The trope theorists adopting a *sparse theory of tropes* (cf. Sec. 1) postulate tropes to determine at least the electric charges of the fundamental particles.[14] First, all tropes of electric charge are mutually related by some relation of positive or negative proportion, which can be expressed by rational numbers. Assume that t_1 is a -e charge trope and t_2 is a -e/3 charge trope. The following statement holds true of tropes t_1 and t_2:

[1]: Trope t_1 is in 3:1 proportion to trope t_2.

Moreover, assuming that t_3 is an e/3 charge trope, the following statement holds true of tropes t_1 and t_3:

[2]: Trope t_1 is in -3:1 proportion to trope t_3.

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[13] These basic units include both the charges of leptons and their anti-particles as well as the charges of quarks and anti-quarks.

[14] According to some trope theorists (e.g. Simons 1994, 1998), only simple objects are trope bundles. By contrast, Keith Campbell (1990) postulates complex tropes, which are properties of complex objects.

Certain metaphysicians, most notably Yuri Balashov (1999), have argued that we need to introduce zero-value quantitative properties to provide the best metaphysical account of certain basic physical quantities, such as electric charge and spin. We must sharply distinguish between having a zero-value quantity (e.g. zero electric charge) and a mere absence of that quantity. Assuming that Balashov is right (a claim whose correctness we will not assess in this paper), trope theory needs to provide an account of the necessary relations in which a zero-value charge trope is involved. Let t_4 be a zero-value charge trope. Let n be some rational number other than zero. The following statement holds:

[3]: Trope t_4 is in 0:n proportion to trope t_3.

Let us call the relation of 0:n proportion *zero proportion*. Claim [3] requires explanation. Since the choice of unit for electric charge is conventional, the absolute quantitative value assigned to trope t_3 does not matter in expressing the relation of zero proportion (or, for the matter, any proportion relation); all that matters is that the proportion remains fixed. Trope t_4 and all other zero-charge tropes are connected by the *asymmetric* relation of zero proportion to all non-zero-charge tropes.

Second, all electric charge tropes are mutually connected by *greater than or equal to*, which we call the relation of order. This relation is a non-strict total order.[15] For instance, the following statement about tropes t_1 and t_3 is true:

[15] In other words, all charge tropes are connected by a *greater than or equal to* relation; the relation is transitive and non-symmetric (neither asymmetric nor symmetric).

[4]: Trope t_3 is greater than or equal to trope t_1.

Here, what is determined by the nature of tropes is the unified direction of the relation of order. By contrast, whether we call this a "lesser than or equal to" or a "greater than or equal to" relation is up to the conventionally chosen unit of electric charge.

There is an obvious connection between these two necessary relations. If the relation of order between two charge tropes holds only in one direction (like in the case of tropes t_1 and t_2, and t_4 and t_3), the tropes are related by some asymmetric relation of proportion (such as 3:1 proportion or zero proportion). By contrast, if the relation of order holds in both directions, the tropes are "exactly similar" or "equal". Two zero-charge tropes t_4 and t_5 are equal in this sense. For obvious reasons, the relation of proportion does not hold between zero-charge tropes (one cannot divide by zero). All mutually equal charge tropes that are not zero-charge tropes are connected by the relation of 1:1 proportion, like two -e charge tropes.

It is now easy to choose some unit for electric charge. We can take any of the electric charges of the charge tropes mutually connected by 1:1 proportion and select it as the unit. A natural choice for the unit would be a charge of some of the basic particles (cf. above) such as positron charge, e.[16] Which values of charge are negative and which positive is stipulated in connection to the choice of unit. By means of the conventionally chosen unit, we can specify quantitative distances ("inexact resemblances") between electric charge tropes – that zero-charge trope t_4 is 1e greater than -e charge trope t_1, for instance.

The relations of proportion and order are *basic internal relations* between electric charge tropes. Consider, for instance, the relation of -3:1 proportion between tropes t_1 and t_3.

[16] That we *call* this quantity "e" and "charge" is, of course, based on our conventions, which presuppose that tropes are in the required relations of proportion and order, cf. below.

First, -3:1 proportion is a proto internal relation: necessarily, t_1 and t_3 are in the relation of -3:1 proportion if and only if t_1 and t_3 exist. Second, the obtaining of the relation of -3:1 proportion is not derivative from proto internal relations holding between entities some of which are distinct from t_1 and t_3. In order to reach this conclusion, it suffices to maintain that tropes t_1 and t_3 are simple particular natures (particular natures with no proper parts).[17] Moreover, given that t_1 is a particular -e charge and t_3 is a particular e/3 charge, they are connected by the relation of -3:1 proportion. We need not postulate any entities distinct from t_1 and t_3. For instance, the holding of this proportion relation is not derivative from the holding of instantiation between tropes t_1 and t_3 and the respective property universals. In other words, -3:1 proportion is not a derived internal relation between tropes t_1 and t_3.[18] Hence, -3:1 proportion is a basic internal relation between tropes t_1 and t_3. In a similar way, we can show that all other relations of proportion and order between charge tropes (e.g. those described by clauses [1], [3] – [4]) fulfil condition [BIR] and are basic internal relations.

The basic internal relations of proportion and order are determined by the nature of tropes. It is a direct consequence of the existence of electric charge tropes that they are in

[17] Thus, we adopt the standard conception of tropes as particular natures defended by Campbell (1990), Maurin (2002, 2005), and Simons (2003). Moreover, as any basic entities of a category system, such as property universals in a system postulating universals, tropes stand in *different formal ontological relations*, which are also basic internal relations, cf. Section 2.

[18] This kind of derived internal relation would fulfil [PIR], [DIR], and [IR2], but not [BIR]. Ellis (2001) and Lowe (2006, 2012) adopt this type view by claiming that all tropes (or modes) instantiate the respective property universals (e.g. that of -e charge). Lowe (2012, p. 412) is also explicit in maintaining that, necessarily, if modes and certain property universals exist, modes instantiate these property universals.

these internal relations. Therefore, the relations of proportion and the relation of order remain invariant irrespective of the conventionally chosen unit of electric charge. How the relation of order is expressed (as a “lesser than or equal to” or “greater than or equal to” relation) is dependent on the conventional choice of the unit.

Electric charge tropes appear to bear proportion-like relations to quantity tropes falling under some other determinable (mass tropes, for instance). Nevertheless, these proportion-like relations are, unlike the internal relations of proportion and order, determined by the conventional choice of units (for mass and electric charge). The dependence of these proportion-like relations on the choice of units can be seen in the fact that they do not remain invariant in *all* changes of the units, for instance, in such changes where we leave the unit of mass constant but choose some other unit for electric charge. Similarly, the proportion-like relations charge tropes appear to bear to entities other than tropes, e.g. numbers, do not remain invariant. Since these proportion-like relations are not determined by the nature of tropes, but rather by the conventional choice of units, they are not basic internal relations.

All electric charge tropes are mutually connected by the relation of order. Moreover, all non-zero-charge tropes are mutually connected by some relation of proportion. Zero-charge tropes, if there are such entities, are all connected by the relation of zero proportion to all non-zero-charge tropes. By means of the relations of proportion and the relation of order, we can specify the falling of a trope under the highest determinable, namely electric charge. The best way to do this is to select some arbitrary non-zero-charge trope *u* as a paradigm (or sample). Trope *t* is a charge trope if and only if *t* bears the relation of order and some relation of proportion to trope *u*. With the help of these basic internal relations, we can account for the falling of trope *t* under a determinable without reference to the determinable, namely electric charge. The falling of tropes under a determinable is determined by their nature, and there is no need to assume determinables as separate entities (e.g. determinable universals). Similarly, we can also specify quantitative distances between all charge tropes without recourse to the determinable.[19]

In order to specify the tropes falling under some specific determinate quantity – say, -e charge – we can proceed in a similar way. We can select any -e charge trope *u* as a paradigm. Trope *t* is a -e charge trope if and only if *t* is greater than or equal to *u* and *u* is greater than or equal to *t*. In other words, the two tropes are equal in order. Which trope *u* is selected as a paradigm does not matter, since the internal relations of order are determined by the nature of the tropes. The paradigm only serves as a "sample" in picking up the right kind of tropes.

It is fairly easy to generalize the present approach to most of the other basic monadic physical quantities, such as spin and rest mass. In these cases, we can show in a similar way that the tropes falling under a determinable are mutually connected by the basic internal relations of proportion and the basic internal relation of order. A minor complication results from the fact that rest masses do not come in natural units and it seems to be consistent with physics that they have arbitrarily small values (Balashov 1999). Different rest masses are not multiples of a common (positive) unit, and we need real numbers to express the relations of proportion between rest mass tropes.

In the present approach, we can specify the highest determinable quantity D under which any given quantity trope *t* falls by means of the relations of proportion and the relation of order. Let *u* be the non-zero quantity trope selected as a paradigm among the D tropes. Trope *t* falls under highest quantitative determinable D if and only if *t* is in some relation of (positive, negative, or zero) proportion and in the relation of order to *u*. The quantity tropes falling under distinct highest determinables are not connected by any relation of proportion or by the relation

[19] We can give quantitative distances only relative to some conventionally chosen unit. But as we saw above, the unit can be given without reference to the determinable.

of order. The different relations of proportion and the relation of order divide quantity tropes into distinct and mutually exclusive pluralities (equivalence classes).[20] Therefore, every trope falls under exactly one highest determinable.

In order to avoid misunderstandings, it is important to emphasize the order of ontological priority in our account. First, we postulate individual tropes, which are particular natures (e.g. determinate electric charges). Second, these tropes stand in the basic internal relations of proportion and order. Third, because of being invariant and determined by the nature of tropes, the relations of proportion and order generate natural divisions among tropes. We need no recourse to the specific quantities (mass, electric charge, etc.) under which the tropes fall. We obviously talk about certain kinds of tropes (e.g. charge tropes), which reflects the way we get to know them. Nevertheless, from the ontic point of view, the division of tropes into determinate/determinable kinds is only a by-product of their existence and standing in the basic internal relations of proportion and order.

Thus, there is a certain kind of uniformity among quantity tropes because any quantity trope bears the relations of proportion and order to *some* other quantity tropes. The relations of proportion and order provide us with a comprehensive account of the "similarities" between quantity tropes falling under a determinable. Nevertheless, the nature of monadic quantity tropes falling under given determinable D is by no means exhausted by the relations of proportion and order they generate. Rather, the nature of these tropes is closely connected to their ability to determine the causal powers of the particles that possess them. For instance, electric charge tropes determine the causal powers of a particle to attract or repel other charged

[20] In saying this, trope theorists may remain uncommitted to entities other than just tropes connected by these basic internal relations.

particles by some determinate force.[21] Tropes falling under distinct determinable quantities differ in the kinds of causal powers they determine.

The present approach still leaves us with open questions. The most important of them is the general applicability of the approach to all monadic quantity tropes. It seems to work well in the case of such monadic physical quantities as masses, electric charges, and spin quantum numbers.[22] Moreover, we remain optimistic about the possibility of generalizing the approach to relational tropes (e.g. the tropes of space-time intervals if there are such entities).[23] Nevertheless, quark colour charges seem to form a unified system of two or three complementary charges. If there are such entities as monadic tropes of colour charge, we need to give a more detailed account of the necessary relations between them.[24] Despite these open issues, the present approach is superior to its main alternatives, as we will argue in the next section.

[21] It might be attractive to consider these determination relations as metaphysically necessary, for instance, that charge tropes in certain relative locations necessarily generate certain kinds of attractive or repulsive forces between objects. One variant of such a view would be a dispositionalist conception of tropes (cf. Whittle 2008). However, the details of such a view need to be worked out.

[22] Cf. Balashov (1999) and Morganti (2009) for useful discussions of the basic monadic physical quantities.

[23] Cf. Maurin (2002, Ch.6; 2011) and Wieland & Betti (2008) for an interesting recent account of relational tropes as relata-specific relations.

[24] Cf. Maudlin (2007, p. 86ff.) for a discussion of colour charge.

4. The superiority of the present approach

The gist of the present approach is that the relations of proportion and order are determined by quantitative property tropes. Given that tropes exist as particular natures, they are internally related in certain ways by the basic internal relations of proportion and order. We need the relations of proportion and order in characterizing the “similarities” between quantity tropes and their belonging to determinate/determinable kinds. We need no additional internal relations. Nevertheless, the tropes falling under the additional empirically discovered determinables (such as colour charge(s)) may require a more elaborate account in the characterization of their being tropes belonging to a determinable kind.

In the account of “similarities” between tropes, the present approach does not rely on the primitive internal relations of exact similarity or inexact resemblance (quantitative distance). According to the currently standard approach to trope similarity (Campbell 1990; Maurin 2002; Simons 2003), the tropes falling under a determinate (e.g. 1 kg tropes) are mutually connected by the relation of exact similarity. According to this approach, exact similarity is a primitive basic internal relation between tropes: given that two distinct 1 kg tropes *t* and *u* exist, they are exactly similar. It is a natural expansion of this view to consider the tropes falling under a determinable inexactly similar. Inexact similarities between quantity tropes allow for a more precise expression in terms of quantitative distances: for instance, 3 kg trope *v* is 1 kg greater than 2 kg trope *w*. The relations of quantitative distance and exact similarity generate a non-strict total order ("equal to or greater than") among the tropes falling under a determinable (e.g. mass tropes).

Our worry with this account concerns quantitative distances. Consider, again, 3 kg trope *v* and 2 kg trope *w*. We must give the quantitative distance between *v* and *w* by recourse to the unit of mass: 3 kg trope *v* is 1kg greater than 2 kg trope *w*. The reference to the unit contains an implicit reference to the determinable (mass): trope *v* is by a certain unit of mass U greater than trope *w*. The advocate of the (extended) standard approach takes this quantitative distance as primitive and not to be accounted for by means of any other internal relation between *v* and *w*. Therefore, she seems to be unable to show that tropes are in relation of quantitative distance without themselves being instances of determinable universals (determinable kinds of tropes).[25]

Hence, it is a general problem for the standard approach that inexact similarities considered as quantitative distances are not good candidates for (primitive) basic internal relations. A precise characterization of these internal relations requires recourse to the determinable with respect to which tropes are inexactly similar. One is inclined to consider tropes connected by these relations as instances of a determinable property. This is in a stark contrast with the conception presented in the previous section, in which “exact similarities” and "inexact resemblances" between quantity tropes are laid down without recourse to any determinable. There is no threat of circularity here because the relations of proportion and order uniquely specify the determinate/determinable quantity under which given tropes fall.

There are two more approaches to inexact resemblances between quantity tropes that merit a brief consideration. Both of them can be developed independently of one’s account of “exact resemblances”. In order to avoid the primitive internal relation of exact resemblance, we may assume that exact resemblances are accounted for by means of the relation of order in a way envisaged in the previous section.

[25] In the present case, one may, e.g. consider quantitative distance a derived internal relation between tropes *v* and w; it holds because *v* and w instantiate the respective determinate kind universals and the kind universals are internally related. Cf. Ellis (2001, Ch.2) for a more detailed view of tropes as instances of determinate and determinable kind universals.

First, Keith Campbell (1990, Secs. 4.3-4.4) considers the quantity tropes falling under a determinable (e.g. mass tropes) as complex tropes, conjunctive compresences formed by the tropes of the smaller units of the same quantity. The basic idea behind this is very simple: two distinct, mutually co-located quantity tropes falling under the same determinable, two mass tropes *t* and *u*, for instance, form a complex trope whose quantitative value amounts to the sum of the values of *t* and *u*. If *t* is a 1 kg trope and *u* is a 2 kg trope that are mutually co-located, they form 3 kg trope *v*. More generally, every aggregate of mutually co-located (compresent) mass tropes constitutes a complex trope, a *conjunctive compresence*, whose quantitative value amounts to the value of the sum of the constituent tropes.

Campbell takes tropes to be independent existents, which can exist without being accompanied by other tropes.[26] Consequently, tropes falling under a determinable can form conjunctive compresences independently of the existence of the tropes falling under any distinct determinables. The theory allows for some quantity tropes (such as electric charge tropes) to be constituted by tropes of some single basic unit (e/3 charge). Hence, it can accommodate the fact many quantities come in natural units. In the context of Campbell's trope theory, we can form a general account of complex tropes in terms of conjunctive compresences.[27]

[26] To be more precise, according to Campbell (1990), every trope *t* exists independently of any other trope that is not a proper part of *t*. The only exceptions of this rule are positions of space-time, which he claims to be separate "quasi-tropes" or "place tropes": every trope *t* must be accompanied by some (although not any specific) place trope.

[27] As a generalization of his conception of conjunctive compresences, Campbell allows for the distinct maximal aggregates of co-located tropes falling under a determinable in different locations to form conjunctive compresences, whose value amounts to the sum of the values of all of their constituent tropes.

Despite its impressive merits, Campbell’s theory has (at least) two serious problems. First, as noted in the previous section, rest masses do not come in natural units but get arbitrarily small values. Moreover, we may well need irrational numbers to present the proportions between distinct rest masses. It seems that we cannot present “larger masses” as multiples of some common smaller unit mass (however small). Therefore, it seems that we cannot reduce all rest mass tropes to conjunctive compresences of rest mass tropes of some common smaller unit. The second problem with Campbell’s theory is that it does not give any account of the inexact resemblances between the tropes of negative and positive units of some quantity (e.g. the tropes of negative and positive charge).[28] Because of these two serious problems, Campbell's account is not a credible competitor of our approach, analysing inexact similarities in terms of the relations of proportion and order.

One can construe another possible approach to inexact resemblances in terms of the necessary relation of order (greater than or equal to) and a primitive three-place summation relation.[29] The latter is supposed to provide quantitative distances between the tropes falling under a determinable. For instance, every 1 kg trope *t* and 2 kg trope *u* bears the summation relation to *any* 3kg trope *w*. A trope whose quantitative value amounts to a sum of lesser values of the quantity need not be considered complex. (This is a major difference between this approach and Campbell's theory.) Rather, the latter bears the primitive summation relation to

[28] However, the advocate of conjunctive compresences may consider the restrictions to the composition of tropes in her theory as brutal. Therefore, she can avoid all reference to determinables in her account of complex tropes.

[29] A similar approach to inexact resemblances between quantitative properties is advocated by Maya Eddon (2013), who follows Mundy (1987). However, she considers quantities as universals and order and summation as second-order relation universals.

the tropes having the lesser values. Therefore, we seem to be able to account for the "inexact similarities" between the fundamental tropes of the negative and positive values of the same quantity (the tropes of negative and positive charge, for instance). Moreover, in giving quantitative distances by means of the summation relation, the approach clearly avoids the postulation of determinable universals.

The main problem of this approach concerns the general assumptions it makes about the existence of different kinds of tropes: assume that *w* is an arbitrary trope greater than trope *t*. Given that these two tropes *t* and *w* exist, there must exist *some* third trope *u* that fulfils the following condition: *t* and *u* bear the three-place summation relation to trope *w*. The quantitative value of *w* amounts to the sum of the values of *t* and *u*. However, if tropes are discovered *a posteriori* as properties of objects, there can be no guarantee that such trope *u* should exist. Rather, this approach is simply committed to an *a priori* assumption that *some* such trope *u* exists.[30] By contrast, if we present "inexact similarities" by means of the relations of proportion and order, we need not commit ourselves *a priori* to the existence of such extra tropes. Hence, although the latest approach has certain interesting features, it is not a serious rival to our approach.

Our account presented in the previous section provides the best currently available conception of "similarities" between quantity tropes. With the help of basic internal relations of proportion and order, we obtain an accurate conception of similarities and eliminate any reference to determinables. Hence, we explicitly eliminate any need for assuming determinable universals.

[30] In constructing her original approach, Eddon (2013) assumes that monadic properties are Platonic universals, which need not be instantiated in order to exist.

5. Conclusion

In this article, we have defended a new conception of internal relations between quantity tropes belonging to determinate and determinable kinds acceptable to the advocates of different trope bundle theories. The core idea behind our conception is that individual tropes are, as determinate particular natures, internally related by certain relations of proportion and order. By being determined by the nature of tropes, the relations of proportion and order give rise to natural divisions among tropes, and divide them into tropes belonging to distinct determinable and determinate kinds.

In Section 2, we presented a novel distinction between different kinds of internal relations with two important new features. First, unlike the standard distinction, our new distinction does not rely on inherence (objects having properties). Instead, we distinguish between *basic* and *derived internal relations*. Basic internal relations constitute a sub-group of proto internal relations, internal relations holding necessarily if their relata exist. The holding of a basic internal relation is not derivative from the holding of additional internal relations connecting entities some of which are distinct from the relata of the original relation. Thus, second, being necessitated by the existence of the related entities is not sufficient for a relation to be a basic internal relation. We also demand that the holding of the relation does not depend (even indirectly) on the existence of any specific entities distinct from its original relata.

Being equipped with this improved account of internal relations, we argued in Section 3 that quantity tropes falling under determinable D are mutually connected by the basic internal relations of proportion and order. Since these basic internal relations hold in virtue of the nature of the related tropes, we can select *any* non-zero-value D trope *u* as a sample (paradigm); trope *t* is a D trope if and only if *t* is connected by the relation of order and some relation of proportion to trope *u*. In this way, we can specify the determinable under which given trope *t* falls by means of the nature of tropes under consideration without recourse to the determinable. In a similar way, we can specify the determinate kind to which a given trope belongs.

In Section 4, we argued that our account is superior to its main recent rivals. The "standard conception" takes exact similarity and inexact resemblances (quantitative distances) as primitive internal relations. The advocates of the standard conception are obliged to give an accurate account of these primitives. It seems that any attempt to present them as primitive internal relations involves reference to a determinable. Therefore, it may be difficult to avoid commitment to determinable universals. The two additional proposals presented in Section 4 do not presuppose the existence of universals. Nevertheless, neither of them turned out to be a serious alternative to our approach: whereas Campbell's proposal is inadequate, the use of the primitive summation relation makes us involved in the existence assumptions entirely unmotivated by the sparse nominalist perspective.

Since we subscribe to the empirically motivated s*parse theory of tropes*, we remain open to the discovery of tropes belonging to new determinable kinds. Moreover, our approach may need to be developed further to be applicable to quantity tropes belonging to such new kinds. We already mentioned a comparatively recently discovered determinable kind(s), namely, quark colour charges, in Section 3. It also falls out of the scope of the present article to defend a trope nominalist theory of determinable and determinate kinds, but our theory is compatible with different alternatives (e.g. considering determinables/determinates as sets of tropes or denying their existence altogether). We must also leave answering the specific arguments presented for the existence of determinate or determinable universals (e.g. referring to the alleged need for universals as truthmakers of laws of nature) for another occasion.

Acknowledgements

We wish to thank Gonzalo Rodriguez-Pereyra, Tuomas Tahko, and two referees of Erkenntnis for helpful comments on earlier versions of this paper. Moreover, we thank Finnish Cultural Foundation for funding this research.

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